

## Section 9.2

### **Check Your Understanding, page 560:**

1. *State:* We want to perform a test at the  $\alpha = 0.05$  significance level of  $H_0 : p = 0.20$  versus  $H_a : p > 0.20$  where  $p$  is the true proportion of all teens at the school who would say they have electronically sent or posted sexually suggestive images of themselves. *Plan:* If conditions are met, we should do a one-sample  $z$  test for the population proportion  $p$ . *Random:* The sample was randomly selected. *10%:* The sample size (250) is less than 10% of the 2800 students at the school. *Large Counts:*  $np_0 = 250(0.2) = 50 \geq 10$  and  $n(1 - p_0) = 250(0.8) = 200 \geq 10$ . *Do:* The sample proportion is  $\hat{p} = \frac{63}{250} = 0.252$ . The corresponding test statistic is

$$z = \frac{0.252 - 0.20}{\sqrt{\frac{0.20(0.80)}{250}}} = 2.06 \text{ and the } P\text{-value is } P(Z \geq 2.06) = 0.0197. \text{ Conclude: Because the } P\text{-}$$

value of 0.0197 is less than  $\alpha = 0.05$ , we reject  $H_0$ . We have convincing evidence that more than 20% of the teens in her school would say they have electronically sent or posted sexually suggestive images of themselves.

### **Check Your Understanding, page 563:**

1. *State:* We want to perform a test at the  $\alpha = 0.05$  significance level of  $H_0 : p = 0.75$  versus  $H_a : p \neq 0.75$  where  $p$  is the true proportion of all restaurant employees at this chain who would say that work stress has a negative impact on their personal lives. *Plan:* If conditions are met, we should do a one-sample  $z$  test for the population proportion  $p$ . *Random:* The sample was randomly selected. *10%:* The sample size (100) is less than 10% of all employees at this large restaurant chain. *Large Counts:*  $np_0 = 100(0.75) = 75 \geq 10$  and  $n(1 - p_0) = 100(0.25) = 25 \geq 10$ .

*Do:* The sample proportion is  $\hat{p} = \frac{68}{100} = 0.68$ . The corresponding test statistic is

$$z = \frac{0.68 - 0.75}{\sqrt{\frac{0.75(0.25)}{100}}} = -1.62 \text{ and the } P\text{-value is } 2P(Z \leq -1.62) = 2(0.0526) = 0.1052. \text{ Conclude:}$$

Because the  $P$ -value of 0.1052 is greater than  $\alpha = 0.05$ , we fail to reject  $H_0$ . We do not have convincing evidence that the true proportion of all restaurant employees at this large restaurant chain who would say that work stress has a negative impact on their personal lives is different from 0.75.

### **Check Your Understanding, page 564:**

1. In the previous Check Your Understanding, we failed to reject the null hypothesis that the proportion of restaurant employees at this chain who say that work stress has a negative impact on their personal lives is 0.75. The confidence interval given in the output includes 0.75, which means that 0.75 is a plausible value for the population proportion that we are seeking. So both the significance test (which didn't rule out 0.75 as the proportion) and the confidence interval (which gave 0.75 as a plausible value) give the same conclusion. The confidence interval, however, gives more information in that it gives a range of plausible values for the population proportion instead of only making a decision about a single value.

**Check Your Understanding, page 569:**

1. A Type II error is more serious for the potato-chip producer. If a Type I error occurred, they would reject a good shipment of potatoes and have to wait to get a new delivery. However, if a Type II error occurred, they would accept a bad batch and make potato chips with blemishes. This might upset consumers and decrease sales. To minimize the probability of a Type II error, choose a large significance level such as  $\alpha = 0.10$ .

2. (a) Increase power. Increasing  $\alpha$  to 0.10 makes it easier to reject the null hypothesis, which increases power.

(b) Decrease power. Decreasing the sample size means we don't have as much information to use when making the decision, which makes it less likely to correctly reject  $H_0$ .

(c) Decrease power. It is harder to detect a difference of 0.02 ( $0.10 - 0.08$ ) than a difference of 0.03 ( $0.11 - 0.08$ ).

**Exercises, page 570:**

9.31 Random: The sample was randomly selected. 10%: The sample size (60) is less than 10% of all students at his large high school. Large Counts:  $np_0 = 60(0.80) = 48 \geq 10$  and

$n(1 - p_0) = 60(0.20) = 12 \geq 10$ . All conditions have been met.

9.32 Random: The sample was randomly selected. 10%: The sample size (100) is less than 10% of the students at a large elementary school. Large Counts:  $np_0 = 100(0.13) = 13 \geq 10$  and

$n(1 - p_0) = 100(0.87) = 87 \geq 10$ . All conditions have been met.

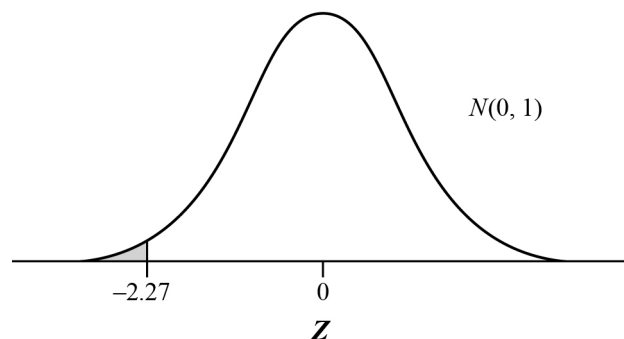
9.33 The expected number of successes  $np_0 = 10(0.5) = 5$  and failures  $n(1 - p_0) = 10(0.5) = 5$  are both less than 10, so the Large Counts condition is not met.

9.34 The expected number of failures  $n(1 - p_0) = 200(0.01) = 2$  is less than 10, so the Large Counts condition is not met.

9.35 (a) In this case  $\hat{p} = 41/60 = 0.683$  so  $z = \frac{0.683 - 0.80}{\sqrt{\frac{0.80(0.20)}{60}}} = -2.27$ .

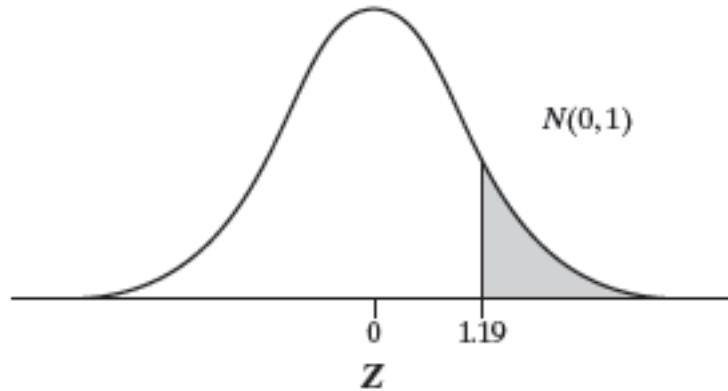
(b) Because this is a one-sided test with  $H_a: p < 0.80$ , the  $P$ -value is  $P(Z \leq -2.27) = 0.0116$ .

Using technology: `normalcdf(lower: -1000, upper: -2.27,  $\mu$ : 0,  $\sigma$ : 1)` = 0.0116. The graph is given below.



9.36 (a) In this case  $\hat{p} = \frac{17}{100} = 0.17$  so  $z = \frac{0.17 - 0.13}{\sqrt{\frac{0.13(0.87)}{100}}} = 1.19$ .

(b) Because this is a one-sided test with  $H_a: p > 0.13$ , the  $P$ -value is  $P(Z \geq 1.19) = 1 - 0.8830 = 0.1170$ . Using technology: `normalcdf(lower: 1.19, upper: 1000,  $\mu$ : 0,  $\sigma$ : 1)` = 0.1170. The graph is given below.



9.37 (a) Because this is a one-sided test, the  $P$ -value is  $P(Z \geq 2.19) = 0.0143$ . 5%: Because the  $P$ -value of 0.0143 is less than  $\alpha = 0.05$ , we reject  $H_0$ . There is convincing evidence that  $p > 0.5$ . 1%: Because the  $P$ -value of 0.0143 is greater than  $\alpha = 0.01$ , we fail to reject  $H_0$ . There is not convincing evidence that  $p > 0.5$ .

(b) If the test were two-sided, the  $P$ -value would be  $2P(Z \geq 2.19) = 2(0.0143) = 0.0286$ . Because this  $P$ -value is still less than  $\alpha = 0.05$  and greater than  $\alpha = 0.01$ , we would again reject  $H_0$  at the 5% significance level and fail to reject  $H_0$  at the 1% significance level.

9.38 Because this is a one-sided test, the  $P$ -value is  $P(Z \leq -1.78) = 0.0375$ . 5%: Because the  $P$ -value of 0.0375 is less than  $\alpha = 0.05$ , we reject  $H_0$ . There is convincing evidence that  $p < 0.65$ . 1%: Because the  $P$ -value of 0.0375 is greater than  $\alpha = 0.01$ , we fail to reject  $H_0$ . There is not convincing evidence that  $p < 0.65$ .

(b) If the test were two-sided, the  $P$ -value would be  $2P(Z \leq -1.78) = 2(0.0375) = 0.075$ . Because this  $P$ -value is greater than both  $\alpha = 0.05$  and  $\alpha = 0.01$ , we would fail to reject  $H_0$  at both the 5% and the 1% significance levels.

9.39 *State:* We want to perform a test of  $H_0 : p = 0.37$  versus  $H_a : p > 0.37$  where  $p$  is the true proportion of all students who are satisfied with the parking situation after the change. We'll use  $\alpha = 0.05$ . *Plan:* If conditions are met, we should do a one-sample  $z$  test for the population proportion  $p$ . *Random:* The sample was randomly selected. *10%:* The sample size (200) is less than 10% of the population of size 2500. *Large Counts:*  $np_0 = 200(0.37) = 74 \geq 10$  and

$n(1 - p_0) = 200(0.63) = 126 \geq 10$ . *Do:* The sample proportion is  $\hat{p} = \frac{83}{200} = 0.415$ . The test

statistic is  $z = \frac{0.415 - 0.37}{\sqrt{\frac{0.37(0.63)}{200}}} = 1.32$  and the  $P$ -value is  $P(Z \geq 1.32) = 0.0934$ . *Conclude:* Because

the  $P$ -value of 0.0934 is greater than  $\alpha = 0.05$ , we fail to reject  $H_0$ . We do not have convincing evidence that the true proportion of all students who are satisfied with the parking situation after the change is greater than 0.37.

9.40 *State:* We want to perform a test at the  $\alpha = 0.05$  significance level of  $H_0 : p = 0.10$  versus  $H_a : p < 0.10$  where  $p$  is the true proportion of the 5000 Alzheimer's patients who would experience nausea while taking the new drug. *Plan:* If conditions are met, we should do a one-sample  $z$  test for the population proportion  $p$ . *Random:* The sample was randomly selected. *10%:* The sample size (300) is less than 10% of the population size (5000). *Large Counts:*

$np_0 = 300(0.10) = 30 \geq 10$  and  $n(1 - p_0) = 300(0.90) = 270 \geq 10$ . *Do:* The sample proportion is

$\hat{p} = \frac{25}{300} = 0.0833$ . The corresponding test statistic is  $z = \frac{0.0833 - 0.10}{\sqrt{\frac{0.10(0.90)}{300}}} = -0.96$  and the  $P$ -value

is  $P(Z \leq -0.96) = 0.1685$ . *Conclude:* Because the  $P$ -value of 0.1685 is greater than  $\alpha = 0.05$ , we fail to reject  $H_0$ . We do not have convincing evidence that the true proportion of these 5000 Alzheimer's patients who would experience nausea while taking the new drug is less than 10%.

9.41 (a) *State:* We want to perform a test of  $H_0 : p = 0.50$  versus  $H_a : p > 0.50$  where  $p$  is the true proportion of boys among first-born children. We will perform that test at the  $\alpha = 0.05$  significance level. *Plan:* If conditions are met, we should do a one-sample  $z$  test for the population proportion  $p$ . *Random:* The sample was randomly selected. *10%:* The sample size (25,468) is less than 10% of all first-borns. *Large Counts:*  $np_0 = 25,468(0.50) = 12,734 \geq 10$  and  $n(1 - p_0) = 25,468(0.50) = 12,734 \geq 10$ . *Do:* The sample proportion is

$\hat{p} = \frac{13,173}{25,468} = 0.5172$ . The corresponding test statistic is  $z = \frac{0.5172 - 0.50}{\sqrt{\frac{0.50(0.50)}{25,468}}} = 5.49$  and the  $P$ -

value is  $P(Z \geq 5.49) \approx 0$ . *Conclude:* Because the  $P$ -value of approximately 0 is less than  $\alpha = 0.05$ , we reject  $H_0$ . There is convincing evidence that first-born children are more likely to be boys.

(b) The results of this study can only be generalized to first-born children because that is the group that we sampled from. In order to generalize to all children we would need to take a random sample of all children, not just first-born children.

9.42 (a) *State*: We want to perform a test of  $H_0 : p = 0.50$  versus  $H_a : p > 0.50$  where  $p$  is the true proportion of coffee drinkers in this small city who prefer fresh-brewed. We will perform the test at the  $\alpha = 0.05$  significance level. *Plan*: If conditions are met, we should do a one-sample  $z$  test for the population proportion  $p$ . *Random*: The sample was randomly selected. *10%*: The sample size (50) is less than 10% of the population of coffee drinkers in this small city. *Large Counts*:  $np_0 = 50(0.50) = 25 \geq 10$  and  $n(1 - p_0) = 50(0.50) = 25 \geq 10$ . *Do*: The sample

proportion is  $\hat{p} = \frac{36}{50} = 0.72$ . The corresponding test statistic is  $z = \frac{0.72 - 0.50}{\sqrt{\frac{0.50(0.50)}{50}}} = 3.11$  and the

$P$ -value is  $P(Z \geq 3.11) = 0.0009$ . *Conclude*: Because the  $P$ -value of 0.0009 is less than 0.05, we reject  $H_0$ . There is convincing evidence that the proportion of coffee drinkers in this small city who prefer fresh-brewed coffee is greater than 0.5.

(b) Because the order that a subject tries the two types of coffee might influence which coffee is preferred, it is important that the order is randomized. For example, if people tend to prefer the most recent sample tasted and we always gave the fresh-brewed coffee second, we wouldn't know if the fresh-brewed was preferred because of the taste or because of the order.

9.43 Here are the errors with corrections:

- The alternative hypothesis is incorrect. It should be:  $H_a : p > 0.75$ .
- The parameter is incorrectly defined. It should be:  $p$  = the true proportion of middle school students who engage in bullying behavior.
- The 10% condition is missing. It should be: the sample size (558) is less than 10% of the population of middle school students.
- The Large Counts condition is incorrect. It should be  $np_0 = 558(0.75) = 418.5 \geq 10$  and  $n(1 - p_0) = 558(0.25) = 139.5 \geq 10$ .
- The test statistic contains four mistakes. The order of subtraction is reversed in the numerator and  $p_0$  should be used in the standard deviation, not  $\hat{p}$ . Also, the sample size should be 558 rather than 445 and be under the radical. The correct test statistic is
 
$$z = \frac{0.7975 - 0.75}{\sqrt{\frac{(0.75)(0.25)}{558}}} = \frac{0.0475}{0.0183} = 2.59.$$
- The  $P$ -value should be for a one-sided test, not a two-sided test. The correct  $P$ -value is  $P(Z \geq 2.59) = 0.0048$ .
- The conclusion has several errors. The  $P$ -value does not measure the probability that the null hypothesis is true. Also, we cannot “prove” a hypothesis is true using a significance test. A correct conclusion is: Because the  $P$ -value of 0.0048 is less than  $\alpha = 0.05$ , we reject  $H_0$ . We have convincing evidence that more than three-quarters of middle school students engage in bullying behavior.

9.44 Here are the error with corrections:

- The hypotheses have several errors. Both hypotheses should be in terms of  $p$ , the true proportion of flips that would land on heads. Also, the null hypothesis should be an equality and the alternative hypothesis should be two-sided. The correct hypotheses are  $H_0 : p = 0.5$  and  $H_a : p \neq 0.5$ .
- What Luisa calls the “10%” condition is actually the “Large Counts” condition.

- What Luisa calls the “Large Counts” condition is actually the 10% condition.
- The test statistic has two errors. It should be a  $z$  test statistic, not a  $t$  test statistic. Also, the order of subtraction is reversed in the numerator. The correct test statistic is

$$z = \frac{0.507 - 0.5}{\sqrt{\frac{0.5(0.5)}{4040}}} = 0.89.$$

- The  $P$ -value is calculated incorrectly. The correct value is  $P\text{-value} = 2P(Z \geq 0.89) = 2(1 - 0.8133) = 2(0.1867) = 0.3734$ .
- The conclusion shouldn't reject  $H_0$  when there is a large  $P$ -value. Also, we can't prove that the null hypothesis is true (the coin is fair), even if the  $P$ -value is large. Here is a correct conclusion: Because the  $P$ -value of 0.3734 is greater than  $\alpha = 0.05$ , we fail to reject  $H_0$ . There is not convincing evidence that Buffon's coin is not balanced.

9.45 *State:* We want to perform a test of  $H_0 : p = 0.60$  versus  $H_a : p \neq 0.60$  where  $p$  is the true proportion of teens who pass their driving test on the first attempt. We will use  $\alpha = 0.05$ . *Plan:* If conditions are met, we should do a one-sample  $z$  test for the population proportion  $p$ . *Random:* The sample was randomly selected. *10%:* The sample size (125) is less than 10% of all teens who take the driving test. *Large Counts:*  $np_0 = 125(0.60) = 75 \geq 10$  and

$n(1 - p_0) = 125(0.40) = 50 \geq 10$ . *Do:* The sample proportion is  $\hat{p} = \frac{86}{125} = 0.688$ . The test statistic is  $z = \frac{0.688 - 0.60}{\sqrt{\frac{0.60(0.40)}{125}}} = 2.01$  and the  $P$ -value is  $2P(Z \geq 2.01) = 2(0.0222) = 0.0444$ .

*Conclude:* Because our  $P$ -value of 0.0444 is less than  $\alpha = 0.05$ , we reject  $H_0$ . There is convincing evidence that the true proportion of teens who pass the driving test on their first attempt is different from 0.60.

9.46 *State:* We want to perform a test of  $H_0 : p = 0.73$  versus  $H_a : p \neq 0.73$  where  $p$  is the true proportion of first-year students at this university who think being very well-off financially is an important personal goal. We will use  $\alpha = 0.05$ . *Plan:* If conditions are met, we should do a one-sample  $z$  test for the population proportion  $p$ . *Random:* The sample was randomly selected. *10%:* The sample size (200) is less than 10% of first-year students at this university. *Large Counts:*  $np_0 = 200(0.73) = 146 \geq 10$  and  $n(1 - p_0) = 200(0.27) = 54 \geq 10$ . *Do:* The sample

proportion is  $\hat{p} = \frac{132}{200} = 0.66$ . The corresponding test statistic is  $z = \frac{0.66 - 0.73}{\sqrt{\frac{0.73(0.27)}{200}}} = -2.23$  and

the  $P$ -value is  $2P(Z \leq -2.23) = 2(0.0129) = 0.0258$ . *Conclude:* Because our  $P$ -value of 0.0258 is less than  $\alpha = 0.05$ , we reject  $H_0$ . We have convincing evidence that the proportion of first-year students at this university that think that being very well-off financially is an important goal is different than 0.73.

9.47 (a) *State:* We want to estimate  $p$  = the true proportion of teens who pass their driving test on the first attempt at a 95% confidence level. *Plan:* We should use a one-sample  $z$  interval for  $p$  if the conditions are met. *Random:* The sample was randomly selected. *10%:* The sample size (125) is less than 10% of all teens who take the driving test. *Large Counts:*  $n\hat{p} = 86 \geq 10$  and  $n(1 - \hat{p}) = 39 \geq 10$ . *Do:* A 95% confidence interval is given by

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 0.688 \pm 1.96 \sqrt{\frac{0.688(0.312)}{125}} = 0.688 \pm 0.081 = (0.607, 0.769).$$

*Conclude:* We are 95% confident that the interval from 0.607 to 0.769 captures the true proportion of teens who pass the driving test on the first attempt.

(b) The DMV's claim is that 60% of teens pass the driving test on their first attempt. Because 0.60 is not in the interval computed in part (a), it is not a plausible value for the true proportion of teens who pass the test on their first attempt. Thus, we have convincing evidence that the true proportion of teens who pass the driving test on their first attempt is different from 0.60.

9.48 (a) *State:* We want to estimate  $p$  = the true proportion of first-year students at this university who think being very well-off financially is an important personal goal at a 95% confidence level. *Plan:* We should use a one-sample  $z$  interval for  $p$  if the conditions are met. *Random:* The sample was randomly selected. *10%:* The sample size (200) is less than 10% of first-year students at this university. *Large Counts:*  $n\hat{p} = 132 \geq 10$  and  $n(1 - \hat{p}) = 68 \geq 10$ . *Do:* A 95% confidence interval is given by

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = 0.66 \pm 1.96 \sqrt{\frac{0.66(0.34)}{200}} = 0.66 \pm 0.066 = (0.594, 0.726).$$

*Conclude:* We are 95% confident that the interval from 0.594 to 0.726 captures the true proportion of first-year students at this university who identify being very well-off financially as an important personal goal.

(b) The national value is that 73% of first-year students think that being very well-off financially is an important goal. Because 0.73 is not in the interval computed in part (a), it is not a plausible value for the true proportion of first-year students at this university who believe this. Thus, we have convincing evidence that the proportion of first-year students at this university who think that being very well-off financially is an important goal is different than 0.73.

9.49 No. Because the value 0.16 is included in the confidence interval, it is a plausible value for the true proportion of U.S. adults who would say they use Twitter. In other words, we do not have convincing evidence that the true proportion of U.S. adults who would say they use Twitter differs from 0.16.

9.50 Yes. Because 0.55 is not included in the confidence interval  $0.59 \pm 0.03 = (0.56, 0.62)$ , it is not a plausible value for the true proportion of U.S. adults who would say that they want to lose weight. In other words, we have convincing evidence that the true proportion of U.S. adults who would say that they want to lose weight differs from 0.55.

9.51 (a) The parameter of interest is  $p$  = the true proportion of U.S. teens aged 13 to 17 who think that young people should wait to have sex until marriage.

(b) *Random:* The sample was randomly selected. *10%:* The sample size (439) is less than 10% of the population of all U.S. teens aged 13 to 17. *Large Counts:*  $np_0 = 439(0.5) = 219.5 \geq 10$  and  $n(1 - p_0) = 439(0.5) = 219.5 \geq 10$ .

- (c) The  $P$ -value is 0.011. Assuming that the true proportion of U.S. teens aged 13 to 17 who think that young people should wait to have sex until marriage is 0.50, there is a 0.011 probability of getting a sample proportion that is at least as different from 0.5 as the proportion in the sample.
- (d) Yes. Because the  $P$ -value of 0.011 is less than  $\alpha = 0.05$ , we reject  $H_0$ . There is convincing evidence that the true proportion of U.S. teens aged 13 to 17 who think that young people should wait to have sex until marriage differs from 0.5.

9.52 (a) The parameter of interest is  $p$  = the true proportion of undergraduates at this large university who would be willing to report cheating by other students.

(b) Random: The sample was randomly selected. 10%: The sample size (172) is less than 10% of all undergraduates at this large university. Large Counts:  $np_0 = 172(0.15) = 25.8 \geq 10$  and  $n(1 - p_0) = 172(0.85) = 146.2 \geq 10$ .

(c) The  $P$ -value is 0.146. Assuming that the true proportion of undergraduates at this large university who would report cheating is 0.15, there is a 0.146 probability of getting a sample proportion that is at least as different from 0.15 as the proportion in the sample.

(d) No. Because the  $P$ -value of 0.146 is greater than  $\alpha = 0.05$ , we fail to reject  $H_0$ . We do not have convincing evidence that the true proportion of undergraduates at this large university who would report cheating differs from 0.15.

9.53 (a) A Type I error would be finding convincing evidence that more than 37% of students were satisfied with the new parking arrangement, when in reality only 37% were satisfied.

*Consequence:* the principal believes that students are satisfied and takes no further action. A Type II error would be failing to find convincing evidence that more than 37% are satisfied with the new parking arrangement, when in reality more than 37% are satisfied. *Consequence:* the principal takes further action on parking when none is needed.

(b) If the true proportion of students that are satisfied with the new arrangement is really 0.45, there is a 0.75 probability that the survey provides convincing evidence that the true proportion is greater than 0.37.

(c) We can increase the power either by increasing the sample size ( $n$ ) or by increasing the significance level ( $\alpha$ ).

9.54 (a) A Type I error would be finding convincing evidence that less than 10% of patients would suffer nausea on the drug, when in reality at least 10% would. *Consequence:* The new drug is used on more Alzheimer's patients, leading to excess nausea. A Type II error would be failing to find convincing evidence that less than 10% would suffer from nausea, when in reality less than 10% would. *Consequence:* The new drug is used on fewer Alzheimer's patients, even though it rarely causes nausea.

(b) If the true proportion of Alzheimer's patients who would experience nausea is really 0.07, there is a 0.54 probability that the results of the study would provide convincing evidence that the true proportion is less than 0.10.

(c) We can increase the power either by increasing the sample size ( $n$ ) or by increasing the significance level ( $\alpha$ ).

9.55 The probability of a Type I error is  $\alpha = 0.05$  and the probability of a Type II error is  $\beta = 1 - 0.78 = 0.22$ .

9.56 The power of the test =  $1 - P(\text{Type II error}) = 1 - \beta = 1 - 0.14 = 0.86$ .



- 9.57 (a) If the true proportion of Alzheimer's patients who would experience nausea is really 0.08, there is a 0.29 probability that the results of the study would provide convincing evidence that the true proportion is less than 0.10.
- (b) To increase the power we need more information. Therefore we need to increase the number of measurements taken ( $n$ ).
- (c) The power will decrease. If we decrease the likelihood of a Type I error, then we increase the likelihood of a Type II error. And if the probability of a Type II error increases, the power will decrease because  $\text{power} = 1 - P(\text{Type II error})$ . Also, if  $\alpha$  is smaller, it becomes harder to reject the null hypothesis. This makes it harder to correctly reject  $H_0$ , reducing power.
- (d) The power will increase. Because 0.07 is further from the null hypothesis value of 0.10, it will be easier to detect a difference between the null value and actual value.

- 9.58 (a) If the true mean is really 5.1, there is a 0.23 probability that the sample will provide convincing evidence that the true mean is different than 5.
- (b) To increase the power we need more information. Therefore we need to increase the number of measurements taken ( $n$ ).
- (c) The power will increase. If we increase the likelihood of a Type I error, then we decrease the likelihood of a Type II error. And if the probability of a Type II error decreases, the power will increase because  $\text{power} = 1 - P(\text{Type II error})$ . Also, if  $\alpha$  is greater, it becomes easier to reject the null hypothesis. This makes it easier to correctly reject  $H_0$ , increasing power.
- (d) The power will increase. Because 5.2 is further from the null hypothesis value of 5.0, it will be easier to detect a difference between the null value and the actual value.

9.59 c

9.60 d

9.61 b

9.62 a

- 9.63 (a) The random variable  $X - Y$  has a Normal distribution with mean  $\mu_{X-Y} = \mu_X - \mu_Y = 4 - 4.2 = -0.2$  and standard deviation  $\sigma_{X-Y} = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sqrt{(0.1)^2 + (0.05)^2} = 0.112$ . This random variable is important because for a CD to fit in the case, it must have a smaller diameter than the diameter of the case. In other words, the variable  $X - Y$  must take on a negative number.
- (b) **Step 1: State the distribution and values of interest.** We want to find  $P(X - Y < 0)$  using the  $N(-0.2, 0.112)$  distribution. **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is  $z = \frac{0 - (-0.2)}{0.112} = 1.79$ . The desired probability is  $P(Z < 1.79) = 0.9633$ . *Using technology:* `normalcdf(lower: -1000, upper: 0,  $\mu$ : -0.2,  $\sigma$ : 0.112)` = 0.9629. **Step 3: Answer the question.** There is a 0.9629 probability that a randomly selected CD will fit in a randomly selected case.
- (c)  $P(\text{all fit}) = (0.9629)^{100} = 0.0228$ . There is a 0.0228 probability that all 100 CDs will fit in their cases.

- 9.64 (a) A completely randomized design would be to take the 10,065 people and randomly assign them to the three treatments, with 3355 people receiving each treatment (\$500 bonus for worker, \$500 bonus for employer, no bonus). After the experiment, compare the mean time to return to work in the three groups or the proportion of each group who get a job within 11 weeks.
- (b) Label the people from 00001 to 10065 in alphabetical order and then take five digits at a time from the table, ignoring repeated numbers and the numbers 10066 to 99999 and 00000. Using Table D starting at line 127, the first 3 people chosen are 06565, 00795 and 08727.
- (c) The purpose of a control group is to give the researchers a baseline for comparison. This will allow them to see if a financial incentive is better than no incentive at all. If there were no control group, the researchers could only make a conclusion about which type of incentive (to the worker or to the employer) is more effective.